PAQ-1603010502020500 Seat No. ______

M. Sc. (Sem. II) (W.E.F. 2016) Examination

August / September - 2020

Physics : CT-05

(Quantum Mechanics-II & Statistical Mechanics)

(New Course)

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions carry equal marks.
(2) Full marks are indicated at the right end of each question.
(3) Symbols have their usual meanings.

1 Answer any seven of the following : **14**

(a) Write the formula for Yakawa potential. Which parameter is considered as a measure of the radius of the atom ?

(b) Compare main features of Born approximation and partial waves.

(c) In the partial wave analysis, the following radial wave equation is obtained :

$$\frac{d^2V_\ell}{dr^2} \pm 2\,ik\frac{dV_\ell}{dr} - \left[U + \frac{\ell(\ell+1)}{r^2}\right]v_\ell = 0$$

Prove that this equation is converted into the following form by considering

$$\frac{d^2V_\ell}{dr^2} << \frac{dv_\ell}{dr}, U \propto \frac{1}{r}, U >> \frac{\ell(\ell+1)}{r^2} \quad \log v_\ell = \mp \frac{i}{2k}\int^r \frac{dr}{r}$$

(d) How from the sign of phase shift $\delta_{\ell,}$ one can predict the nature of the potential ? Explain in brief.

(e) What is the energy surface of energy E defined by $\Im d(p,q) = E$ in $\wp$-space ?

(f) What is partition function ? Write its formula. What is β and h in this formulation.

(g) In the grand canonical ensemble, if

$$\mu = a(v) - v\frac{\partial a(v)}{\partial v}, P = -\frac{\partial a(v)}{\partial v}$$ then prove that -

$$\frac{\partial P}{\partial \mu} = \frac{1}{v},$$ here $v = \frac{V}{N}$.

(h) Write postulates of Quantum Statistics.

(i) What is superfluid ? Explain in brief crawling of liquid helium.

(j) In Ising model, the following equation is derived,

$$E_I\{S_i\} = - \in \sum_{\langle ij \rangle} S_i S_j - H \sum_{i=1}^{N} S_i$$

What the quantities E_I, $\in$ and H indicate ? How $\in$ indicates ferromagnetism or antiferromagnetism ?

2 Answer any two of the following :

(a) In the scattering theory, by adopting the wave mechanical approach obtain the following result, **7**

$$\frac{d\sigma(\theta, \phi)}{d\Omega} = |f(\theta, \phi)|^2$$

(b) Define Born approximation and obtain that **7**

$$F_B(\theta) = K^{-1} \int_0^\infty r \sin kr \, U(r) dr$$

(c) In the validity of Born approximation use the **7**

following formula, $\frac{m}{\hbar^2 k} \left| \int_0^\infty \left(e^{2ikr} - 1\right) V(r) dr \right| << 1$ to apply in the case of square well potential of depth V_0 and range a and derive the following result -

$\frac{mV_0}{2\hbar^2 k^2} \left(\rho^2 - 2\rho \sin \rho - 2\rho 2 \cos \rho + 2\right)^{1/2} << 1$. Where, $\rho = 2Ka$, further show that for $\rho << 1$, this expression is

$$\approx \frac{mV_0}{2\hbar^2 k^2} \left(\frac{1}{2}\rho^2\right).$$

3 (a) For the partial wave analysis show that the $f(\theta)$ is a sum of contributions from partial waves from $\ell = 0$ to ∞, and derive the relation : 7

$$f(\theta) = K^{-1} \sum_{\ell=0}^{\infty} (2\ell + 1) e^{i\delta l} \sin \delta_{\ell} P_{\ell}(\cos\theta)$$

(b) In the partial wave analysis derive the following expression for Born approximation for phase shift, 7

$$\sin \delta_{\ell} = -K \int_{0}^{\infty} U(r) r^2 j_{\ell}^2 (kr)\, dr .$$

OR

3 (a) For classical ideal gas, obtain the following expression : 7

$$E = \left(\frac{3}{4\pi} \frac{h^2}{m} \right) \frac{N}{V^{2/3}} \exp\left(\frac{2}{3} \frac{S}{NK} - 1 \right)$$

(b) The energy fluctuations in the canonical ensemble show that the canonical ensemble is Mathematically equivalent to micro-canonical ensemble. Do this by adoping the average energy approach and show that - 7

$$\left\langle H^2 \right\rangle - \langle H \rangle^2 = KT^2 C_r .$$

What happens if $N \to \infty$?

4 Answer any two of the following :

(a) In the grand canonical ensemble derive the following relation, 7

$$\rho(p, q, N) = \frac{z^N e^{-\beta VP - \beta \mathfrak{g}(p,q)}}{N!\, h^{3N}}$$

(b) Explain micro-canonical ensemble in quantum statistics. 7

(c) In the Ising model considering the nearest pair interaction such as $(+,+)(+,-),(-,-)$ with their numbers as N_{++}, N_{+-}, N_{--} respectively. Draw the diagram showing interactions and derive the following interaction energy, **7**

$$E_I\left(N_+, N_{++}\right) = -4 \in N_{++} - \left(-\frac{1}{2}r \in -H\right) N + 2(\in r - H) N_+ .$$

5 Write any two notes :

(a) Born series. **7**

(b) Gibbs paradox. **7**

(c) $\lambda -$ transition. **7**

(d) Eikonel approximation. **7**